A METHOD OF CALCULATING THE HEAT TRANSFER IN A LONGITUDINALLY IRRIGATED ROD BUNDLE

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A method for calculating the heat transfer from a gas to a longitudinal rod bundle with uneven heating along the length and around the periphery is given.

Longitudinal flow around tubes or rods is used in various novel exchangers, which has given considerable interest to the local heat-transfer conditions and frictional resistance in such devices.

Here we consider a method of examining the heat transfer in a channel of complicated cross section, including one represented by a bundle of rods (Fig. 1), where the coolant flows with Pr ~ 1. This constitutes an extension of concepts used for annular channels [1], i.e., we extend the analytical method based on superposition of fundamental solutions [1, 2] to rod bundles with uneven heat production around the perimeter.

The method is applicable to experimental and theoretical studies, since the fundamental solution, as in the case of turbulent flows in annuli [3], are determined by experiment.

We make the following assumptions, which have been used elsewhere [3, 4] for turbulent flow in channels:

1) The flow and heat transfer are quasistationary, i.e., the average characteristics do not vary in time;

2) the heat produced by dissipation of the kinetic energy is negligible (this is true for a coolant speed much less than the speed of sound):

3) the effects of mass forces are small by comparison with those of forces due to viscosity and inertia;

4) the variations in heat-flux density along the axis due to heat conduction and turbulent transport are samll by comparison with the radial variations;

5) there are no internal heat sources in the flow;

6) the region is far from the initial section.

The superposition principle is applicable to an object described by linear differential equations, so we assume that the thermophysical and transport properties of the coolant and constructional materials vary little with temperature, although they may take any values provided that they are sufficiently smooth functions of the spatial coordinates.

Then the heat transfer in a rod bundle is described by the following equation [5]:

$$c_p(r,\varphi,z)\rho(r,\varphi,z)\left[\omega_r(r,\varphi,z)\frac{\partial t_l}{\partial r}+\frac{\omega_\varphi(r,\varphi,z)}{r}\frac{\partial t_l}{\partial\varphi}+\omega_z(r,\varphi,z)\frac{\partial t_l}{\partial z}\right]=$$

$$=\frac{1}{r}\cdot\frac{\partial}{\partial r}\left[r(\varepsilon_t+\lambda_l)\frac{\partial t_l}{\partial r}\right]+\frac{1}{r^2}\cdot\frac{\partial}{\partial\varphi}\left[(\varepsilon_{t\varphi}+\lambda_l)\frac{\partial t_l}{\partial\varphi}\right], \quad (1)$$

$$\frac{1}{r}\cdot\frac{\partial}{\partial r}\left(r\lambda_w\frac{\partial t_w}{\partial r}\right)+\frac{1}{r^2}\cdot\frac{\partial}{\partial\varphi}\left(\lambda_w\frac{\partial t_w}{\partial\varphi}\right)+q_v(r,\varphi,z)=0. \quad (2)$$

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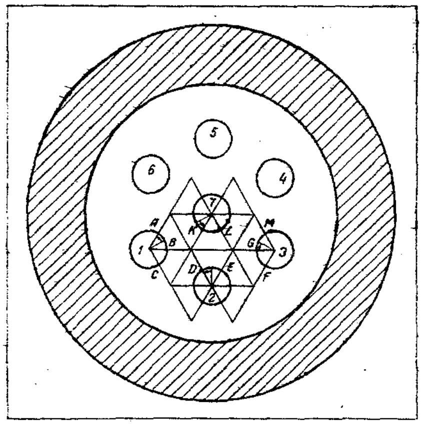

Fig. 1. Bundle of rods in a circular container.

Linkage conditions for the heat fluxes and temperatures in general have to be specified at the wall—coolant boundary:

$$\lambda_l \frac{\partial t_l\,(r,\,\varphi,\,z)}{\partial r}\bigg|_{r=R_i} = \lambda_w \frac{\partial t_w(r,\,\varphi,\,z)}{\partial r}\bigg|_{r=R_i}, \tag{3}$$

$$t_l\,(r,\,\varphi,\,z)|_{r=R_i} = t_w(r,\,\varphi,\,z)|_{r=R_i}. \tag{4}$$

If there are symmetry surfaces in this system for a given distribution of the heat loading, we need examine the heat transfer only in part of the system by setting the heat fluxes equal to zero at the symmetry surfaces:

$$\frac{\partial t_l}{\partial \mathbf{n}}\bigg|_{\Gamma} = 0, \tag{5}$$

where $\mathbf{n}$ is the normal to a symmetry surface Γ.

The two limiting cases are $\lambda_w \gg \lambda_l$ and $\lambda_w \ll \lambda_l$, in which case system (1)-(5) degenerates to one describing the temperature distributions with boundary conditions of the first and second kinds, respectively [6], for an infinite system of uniformly heated rods:

$$t_l\,(r,\,\varphi,\,z)|_{r=R_i} = f_{1i}\,(z), \tag{6}$$

$$-\lambda_l \frac{\partial t_l}{\partial r}\bigg|_{r=R_i} = f_{2i}\,(z), \tag{7}$$

where $f_{1i}(z)$ and $f_{2i}(z)$ are certain functions for each rod i; for instance, $f_{2i}(z) \sim \sin(\pi z/H)$ in a reactor without end reflectors (H is the length of the fuel rods), whereas $f_{1i}(z) = \text{const}\cdot z$ for the case of a boundary condition of the first kind for stabilized heat transfer.

There are major difficulties in solving (1) and (2) subject to the boundary conditions (6) and (7), and even greater difficulties occur for the conjugate problem, which in part is due to the complexity of describing the turbulent flow of a liquid in such a system. Quantitative evaluation of the heat transfer for a rod bundle is possible only if numerical methods and computers are employed [7-11].

The boundary condition (6), which applies for $\lambda_w \gg \lambda_l$, is obeyed for flow of a gaseous coolant in the central cells of a uniformly heated bundle of metal rods or tubes. For instance, air flowing in a bundle of rods of steel 1Kh18N9T at 100 atm and 300-1500°K gives λ_w/λ_l falling from 480 to 350, the corresponding figures for helium under these conditions being from 90 to 70. In the limiting case $\lambda_w \gg \lambda_l$, the heat flux $\lambda_l(\partial t_l/\partial r)|_{r=R_i}$ is most inhomogeneous at the perimeter, which must be borne in mind, since the practical purpose of studies on such systems is to find the temperature distribution for a given distribution of the heat production, i.e., to solve a boundary-value problem with boundary conditions of the second kind, not the first.

When (1) and (2) have been reduced to dimensionless form, we neglect the mixing between channels and obtain

$$\frac{1}{\xi}\cdot\frac{\partial}{\partial\xi}\left[\xi\Lambda(\xi,\varphi,\eta)\frac{\partial\Theta_l}{\partial\xi}\right]+\frac{1}{\xi^2}\cdot\frac{\partial}{\partial\varphi}\left[\Lambda(\xi,\varphi,\eta)\frac{\partial\Theta_l}{\partial\varphi}\right]=W_z\frac{\partial\Theta_l}{\partial\eta}, \tag{1a}$$

$$\frac{1}{\xi}\cdot\frac{\partial}{\partial\xi}\left[\xi\Lambda_w(\xi,\varphi,\eta)\frac{\partial\Theta_w}{\partial\xi}\right]+\frac{1}{\xi^2}\cdot\frac{\partial}{\partial\varphi}\left[\Lambda_w(\xi,\varphi,\eta)\frac{\partial\Theta_w}{\partial\varphi}\right]=Q. \tag{1b}$$

The boundary conditions are formulated by dividing the surface of the rods into elementary heat-transfer parts (segments), whose size and disposition we chose such that the thermal influence of rod No. 1 on the adjacent rod No. 7 occurs entirely within the AB segment, which in our case is 60° (Fig. 1). We assume that the following boundary conditions of the second kind apply for an elementary heat-transfer section:

$$\overline{q_{ij}}|_{-30°\leqslant\psi\leqslant 30°}=\text{const}=\frac{1}{\pi/3}\int\limits_{-\pi/6}^{\pi/6}q_{ij}(\psi)\,d\psi. \tag{8}$$

We introduce fundamental solutions Φ_{ij} analogous to those used in [1, 2] for the system of equations (1a) and (1b), which describes the temperature distribution in a system subject to the boundary conditions (8) for any rod j, whereas the conditions are adiabatic at the surfaces of the other n − 1 rods. For convenience in handling the solutions in determining the temperature distribution in rod i, we introduce subscript i instead of the normal coordinates ξ and φ. The solution for the temperature distribution in this rod is the superposition of a certain number of fundamental solutions if the heat load has an arbitrary distribution over the perimeter of the rods. The temperature distribution in rod i is described by the following equation:

$$\Theta_i=\sum_{j=1}^{n}\frac{1}{\psi_j^{t.s}}\int\limits_{-\psi_j^{t.s}/2}^{\psi_j^{t.s}/2}\Phi_{ij}(\eta,\psi_j)\frac{q_{ij}(\psi_j)}{\overline{q_{c7}}}\,d\psi_j\simeq\sum_{j=1}^{n}\frac{1}{\psi_j^{t.s}}\sum_{k=1}^{m}\Phi_{ij}^{(k)}(\eta,\psi_j)\frac{q_{ij}^{(k)}}{\overline{q_{w7}}}\Delta\psi^{(k)}, \tag{9}$$

where ψ is an angle in the local coordinate system of rod j, whose origin is the line joining that rod to the central rod, while that for the central rod is the line joining it to rod No. 1. If the heat sources are unevenly distributed along a rod, i.e., $q_j = q_j(\eta)$, the resultant temperature distribution is given by Duhamel's relation:

$$\Theta_i=\sum_{j}\frac{1}{\psi_j^{t.s}}\int\limits_{0}^{\eta}\int\limits_{-\psi_j^{t.s}/2}^{\psi_j^{t.s}/2}\Phi_{ij}[(\eta-s),\psi_j]\frac{1}{\overline{q_{w7}}}\cdot\frac{\partial q_{ij}}{\partial s}\,d\psi_j ds, \tag{10}$$

where the integral with respect to s has the meaning of a Riemann–Stieltjes integral.

Here we examine the nonuniformity in the local temperature distributions that become particularly prominent when there are unheated separators and enclosing shells around closely spaced rods ($a/d_h \lesssim 1.2$), in which case it is sufficient to consider the thermal interaction between adjacent rods alone. It is necessary to incorporate the effects of more remote rods as a/d_h increases. Figure 1 shows that rod No. 2 is influenced only by segments CB of rod No. 1, KL of rod No. 7, and GF of rod No. 3. The elementary heat-transfer parts can usually be selected in such a way that the specific heat flux averaged over a part is the same as the heat flux averaged over the entire perimeter, in which case the explicit form of (9) becomes much simpler. A difference from previous studies [2, 3] is that the fundamental solution for the temperature distribution over the perimeter when only one rod is heated is here defined as a trigonometric series:

$$\Phi_{ij}=\sum_{l=0}^{\infty}a_l^{ij}\cos l\psi. \tag{11}$$

The coefficients in (11) are derived from measurements on the temperature variation at the surface and the heat-transfer conditions for this geometry.

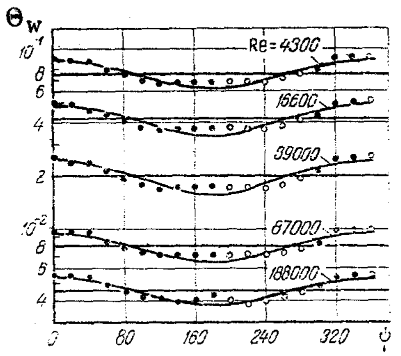

Fig. 2

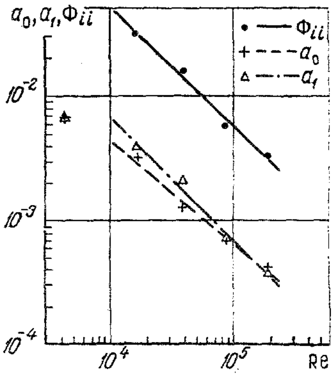

Fig. 3

Fig. 2. Comparison of calculated and measured surface temperatures for peripheral rods [solid line) from theory; filled points) from experiment; open circles) symmetrical continuation of the measurements]; ψ, deg.

Fig. 3. Approximation of the Re dependence of coefficients in the fundamental solutions.

In the case of a gas coolant, the function representing the influence of a rod on itself is independent of ψ, i.e., $\Phi_{ii} \neq F(\psi)$, so the major components of Φ_{ij} will be $l = 0$ and $l = 1$, i.e.,

$$\Phi_{ij} = a_0 + a_1 \cos\psi. \tag{11a}$$

If there is an unheated container,

$$\begin{aligned}\Phi_{ij}|_{\psi=0^\circ} &= a_0 + a_1 = \Theta_{\max},\\ \Phi_{ij}|_{\psi=180^\circ} &= a_0 - a_1 = \Theta_{\min}.\end{aligned} \tag{12}$$

The meanings of the coefficients then become clear:

$$a_0 = 1/2\cdot(\Theta_{\max} + \Theta_{\min}),\ a_1 = 1/2\cdot(\Theta_{\max} - \Theta_{\min}). \tag{13}$$

The coefficients in the fundamental solutions are functions of Re and the rod-bundle pitch a/d_h.

We now envisage simultaneous heating of all rods in the bundle shown in Fig. 1; the temperature of the central rod is then

$$\Theta_7 = \frac{1}{2\pi}\sum_{k=1}^{6}\Phi_{77}\frac{\overline{q_{77}^{(k)}}}{\overline{q_{w^7}}}\cdot\frac{\pi}{3} + \frac{1}{\pi/3}\sum_{j=1}^{6}\Phi_{7j}\frac{\overline{q_{7j}}}{\overline{q_{w^7}}}\cdot\frac{\pi}{3} = \Phi_{ii} + \sum_{j=1}^{6}\frac{\overline{q_{7j}}}{\overline{q_{w^7}}}\left[a_0 + a_1\cos\left(\psi - \frac{\pi}{3}(j-1)\right)\right]. \tag{14}$$

The temperature of a peripheral rod, e.g., No. 2, is

$$\begin{aligned}\Theta_2 &= \frac{1}{2\pi}\sum_{k=1}^{6}\Phi_{22}\frac{\overline{q_{22}^{(k)}}}{\overline{q_{w^7}}}\cdot\frac{\pi}{3} + \frac{1}{\overline{q_{w^7}}}(\overline{q_{21}}\Phi_{21} + \overline{q_{27}}\Phi_{27} + \overline{q_{23}}\Phi_{23}) =\\ &= \frac{1}{2\pi}\sum_{k=1}^{6}\Phi_{ii}\frac{\overline{q_{22}^{(k)}}}{\overline{q_{w^7}}}\cdot\frac{\pi}{3} + \frac{1}{\overline{q_{w^7}}}\left\{\overline{q_{21}}\left[a_0 + a_1\cos\left(\psi - \frac{2\pi}{6}\right)\right] +\right.\\ &\left.+ \overline{q_{23}}\left[a_0 + a_1\cos\left(\psi - \frac{10\pi}{6}\right)\right] + \overline{q_{27}}(a_0 + a_1\cos\psi)\right\}.\end{aligned} \tag{15}$$

If all the rods are uniformly heated,

$$\begin{aligned}\Theta_7 &= \Phi_{ii} + 6a_0,\\ \Theta_2 &= \Phi_{ii} + 3a_0 + 2a_1\cos\psi.\end{aligned} \tag{16}$$

TABLE 1. Errors in Calculations from (16) and (17) for the Surface Temperatures of Peripheral Rods

Re	Dev. of calc. temp. from meas. value, %		Max. dev., %, of meas. temp. from av. over perim.	
	standard deviation	maximum value	positive	negative
4300	3,8	+6,3	19,9	—14,0
16600	4,6	—7,1	24,1	—14,7
39000	5,4	—8,7	25,9	—16,1
87000	4,3	+7,5	21,0	—12,9
188000	4,3	—8,9	19,8	—14,4

We used experimental data [12] to determine a_0, a_1, and Φ_{ii}; the perimeter temperature for the peripheral rods (Fig. 2) was determined by experiment in [12] as the distance $z/d_h = 81.5$ from the inlet to the heated part, and the results are described closely over a wide range in Re by these equations, with the measured values at the perimeter varying within limits of ±20% from the mean temperature. Table 1 gives the errors in using (16) and (17) for the surface temperatures of peripheral rods, as well as the relative deviations of the measured temperatures from the mean over the perimeter for several different Re.

The fundamental solutions are linear logarithmic functions of Re for fully developed turbulence (Fig. 3), and the coefficients obtained by least-squares fitting are

$$\Phi_{ii} = 384\,\mathrm{Re}^{-0.963},$$
$$\Phi_{ij} = 12.3\,\mathrm{Re}^{-0.856} + 75.9\,\mathrm{Re}^{-1.01}\cos\psi. \tag{17}$$

Note that (14)-(17) apply for uniform distribution of the mass flow over the elementary channels.

These equations can be used to calculate the heat transfer in longitudinally irrigated rod bundles in which the thermal loading varies over the perimeter. The method also enables one to determine the rod wall temperature for any distribution of the heat loading and to predict the temperature distribution for cases where experiments have not been performed.

NOTATION

c_p, heat capacity of heat carrier; ρ, density of heat carrier; λ_l, λ_w, thermal conductivities of liquid and wall, respectively; ε_t, turbulent thermal conductivity; ω_r, ω_φ, ω_z, velocity components; $\overline{\omega}$, mean velocity; $W_z = \omega_z/\overline{\omega}$, dimensionless velocity; r, φ, z, coordinates in cylindrical system; ψ_i, angle in local coordinate system of the i-th rod; t_l, t_w, temperatures of liquid and wall, respectively; $\Theta = (t - t_{in})\lambda_l/\overline{q_w} d_h$, dimensionless temperature; d_h, hydraulic diameter; $Q = -q_v d_h/\overline{q_w}$, dimensionless density of volume heat production; $\psi_f^{t.s}$, elementary heat-transfer section on the i-th rod; q_v, heat-production density; q_{wi}, density of heat flux from the surface of the i-th rod; q_{ij}, density of heat flux to the i-th rod from an elementary heat-transfer section on the j-th rod; R_i, surface coordinates of the i-th rod; $\xi = r/d_h$, dimensionless radius; $\eta = 1/\mathrm{Pe}\cdot z/d_h$, dimensionless length; $\Lambda_w = \lambda_w/\lambda_l$, $\Lambda = 1 + \varepsilon_t/\lambda_l$, dimensionless thermal conductivities of wall and liquid, respectively; Pe, Peclet number; Re, Reynolds number.

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DESIGN OF A HEAT PIPE WITH SEPARATE CHANNELS FOR VAPOR AND LIQUID

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The design of a limited rate heat pipe with individual channels for vapor and liquid is discussed.

One of the efficient structures for low-temperature heat pipes to transmit heat in the direction of the gravity field is the heat pipe with separate channels for vapor and liquid — the antigravity heat pipe (AGHP) [1, 2]. The complexity of the physical processes occurring in this type of heat pipe has been an obstacle to a rigorous analytical description of its operation.

In this paper we describe the calculation of the heat-transfer capability and definition of the conditions of operation of an AGHP operating in the evaporation regime (Fig. 1). The input data for the design are the "height" of the heat pipe, its geometrical dimensions, the characteristics of the capillary-porous structure, and also the temperatures of the vapor and condensate being supplied. In the calculation we define the maximum allowable heat-flux surface density in the evaporator and the wall temperature of the compensating cavity, and we verify the condition for boiling of liquid in the cavity.

Under the limiting heat load the capillary heat is equal to the sum of the pressure drops in the individual sections of the heat pipe. The basic equation for a heat pipe of this construction has the form

$$Q\frac{128\eta'' L_{\text{ve}}}{\pi l\rho'' d_{\text{ve}}^4 n_{\text{ve}}} + Q^2\frac{8\Lambda L_{\text{vc}}}{\pi^2 l^2 \rho'' d_{\text{vc}}^5} + Q\frac{128\eta' L_{\text{lc}}}{\pi l\rho' d_{\text{lc}}^4} + Q\frac{a}{\Pi r_{\text{p}}^2}\frac{\eta'}{2\pi l\rho' L_{\text{ve}}}\left(\ln\frac{R_2}{R_1} + 2\ln\frac{R_2+d_{\text{ve}}}{R_2} + \frac{\pi d_{\text{ve}}}{(R_3-R_2-d_{\text{ve}})n_{\text{ve}}}\right) + \rho' gL\sin\varphi = \frac{\xi\sigma\cos\theta}{r_{\text{p}}}. \quad (1)$$

The first term on the left-hand side of Eq. (1) is the pressure drop in the vapor channels of the evaporator; the second term is the drop in the main vapor channel; the third term is the drop in the liquid channel; the fourth term is the friction loss during motion of liquid along the support wall of the wick; the fifth term is the same for the motion of the liquid through the contractions between the vapor channels of the evaporator; and the sixth term is the loss in the wall layer of the evaporator. The complex geometry of the last two sections in this design is approximated by a simpler structure. The wick section from R_2 to $R_2 + d_{\text{ve}}$ between the vapor-emission channels is approximated by an annular layer with the same radii. Contraction of the section carrying liquid is taken into account by an appropriate coefficient (in this case equal to 2), which is obtained by optimizing the number of vapor-emission channels. The wall section for each vapor-emission channel is approximated by a rectangular section with an average length equal to d_{ve} and an average width of $R_3 - R_2 - d_{\text{ve}}$, equal to the thickness of the wall layer. The quantity $\Pi r_{\text{p}}^2/a$ is the perme-

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